

Quantum information processing and multiatom entanglement engineering with a thermal cavity

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We propose a scheme for realizing two-qubit quantum phase gates with atoms in a thermal cavity. The photon-number dependent parts in the evolution operator are canceled with the assistant of a strong classical field. Thus the scheme is insensitive to the thermal field. In the scheme the detuning between the atoms and the cavity is equal to the atom-cavity coupling strength and thus the gates operate at a high speed, which is also important in view of decoherence. The scheme can be generalized to generate multiatom entangled states with a thermal cavity.

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Recently, much attention has been paid to the quantum computers, which are based on the fundamental principles of quantum mechanics. The new type of machines can solve some problems much faster than the classical computers, such as factorizing a large integer [1] and searching for an item from a disordered system [2]. It has been shown that the building blocks of quantum computers are two-qubit gates [3]. In cavity QED, schemes have been proposed for realizing quantum logic gates [3,4]. The ion trap is also a good system for quantum information processing [5]. Quantum logic gates have been demonstrated in cavity QED [6,7], ion trap [8], and NMR [9] experiments.

One of the main obstacles for the implementation of quantum information in microwave cavity QED is the decoherence of the cavity field, while that in ion traps is the difficulty to control the collective vibrational motion of the ions. Recently, Schemes have been proposed for realizing quantum computation in ion traps via virtual vibrational excitations [10,11]. The schemes does not use the motional mode as the data bus and is insensitive to the vibrational states. We have proposed a scheme for generation of two-atom entangled states within a nonresonant microwave cavity [12,13]. The scheme is insensitive to the thermal field and photon decay although the effective Hamiltonian involves photon-number dependent Stark shifts, which are canceled if one atom is initially in the excited state and the other in the ground state. Following our scheme, an experiment has been reported, in which two Rydberg atoms crossing a nonresonant cavity are entangled by coherent energy exchange [14].

In Ref. [12] we also proposed a scheme for the realization of quantum logic gates and atomic state teleportation. In this case a third auxiliary atomic state is employed so that the photon-number dependent Stark shifts can not be canceled. Therefore, the scheme requires that the cavity remain in the vacuum state throughout the procedure and thus is insensitive to the photon decay. However, it is sensitive to the thermal field, which builds up during the operations and relaxes to thermal equilibrium with the increase of the number of qubits and

the number of operations [14]. Thus, the scheme of Ref. [12] is not feasible when scalability is required. In this paper we propose an alternative scheme for realizing two-qubit phase gates in cavity QED. The distinct feature of the present scheme is that the photon-number dependent parts in the evolution operator are canceled with the assistant of a strong classical driving field. Due to this feature the scheme is insensitive to the thermal field. Unlike the previous scheme [12], the present scheme does not require the detuning between the atoms and the cavity to be much larger than the atom-cavity coupling strength and thus the operation speed is greatly improved, which is also important in view of decoherence. Due to these advantages our scheme may open promising prospects for complex quantum information manipulation. The scheme can also be used to generate multiatom entangled states with a single thermal cavity.

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity field and driven by a classical field. In the rotating-wave approximation, the Hamiltonian is (assuming $\hbar = 1$) [15]

$$H = \omega_0 S_z + \omega_a a^\dagger a + \sum_{j=1,2} \left[\frac{g}{2} (a^\dagger S_j^- + a S_j^+) + \frac{\Omega}{2} (S_j^+ e^{-i\omega t} + S_j^- e^{i\omega t}) \right], \quad (1)$$

where $S_j^+ = |e_j\rangle\langle g_j|$, $S_j^- = |g_j\rangle\langle e_j|$, $S_z = \frac{1}{2} \sum_{j=1,2} (|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, with $|e_j\rangle$ and $|g_j\rangle$ ($j=1,2$) being the excited and ground states of the j th atom, a^\dagger and a are the creation and annihilation operators for the cavity mode, and g is the atom-cavity coupling strength, Ω is the Rabi frequency of the classical field, ω_0 is the atomic transition frequency, ω_a is the cavity frequency, and ω is the frequency of the classical field. Assume that $\omega_0 = \omega$. Then the interaction Hamiltonian, in the interaction picture, is

$$H_i = \sum_{j=1,2} \left[\frac{g}{2} (e^{-i\delta t} a^\dagger S_j^- + e^{i\delta t} a S_j^+) + \frac{\Omega}{2} (S_j^+ + S_j^-) \right], \quad (2)$$

$\delta = \omega_0 - \omega_a$. The free Hamiltonian $\omega_0 S_z + \omega_a a^\dagger a$ is used for the transformation to the interaction picture. Define the new atomic basis [11,15]

$$|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle), \quad |-_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle). \quad (3)$$

Then we can rewrite H_i as

$$H_i = \sum_{j=1,2} \left\{ \frac{g}{2} [e^{-i\delta t} a^+ (\sigma_{z,j} + \frac{1}{2}\sigma_j^+ - \frac{1}{2}\sigma_j^-) + e^{i\delta t} a (\sigma_{z,j} + \frac{1}{2}\sigma_j^- - \frac{1}{2}\sigma_j^+)] + \Omega \sigma_{z,j} \right\}, \quad (4)$$

where $\sigma_{z,j} = \frac{1}{2}(|+_j\rangle\langle+_j| - |-_j\rangle\langle-_j|)$, $\sigma_j^+ = |+_j\rangle\langle-_j|$ and $\sigma_j^- = |-_j\rangle\langle+_j|$.

The time evolution of this system is decided by Schrödinger's equation:

$$i \frac{d|\psi(t)\rangle}{dt} = H_i |\psi(t)\rangle. \quad (5)$$

Perform the unitary transformation

$$|\psi(t)\rangle = e^{-iH_0 t} |\psi'(t)\rangle, \quad (6)$$

with

$$H_0 = \Omega \sum_{j=1,2} \sigma_{z,j}. \quad (7)$$

Then we obtain

$$i \frac{d|\psi'(t)\rangle}{dt} = H'_i |\psi'(t)\rangle, \quad (8)$$

where

$$H'_i = \sum_{j=1,2} \frac{g}{2} [e^{-i\delta t} a^+ (\sigma_{z,j} + \frac{1}{2}\sigma_j^+ e^{i\Omega t} - \frac{1}{2}\sigma_j^- e^{-i\Omega t}) + e^{i\delta t} a (\sigma_{z,j} + \frac{1}{2}\sigma_j^- e^{-i\Omega t} - \frac{1}{2}\sigma_j^+ e^{i\Omega t})]. \quad (9)$$

Assuming that $\Omega \gg \delta, g$, we can neglect the terms oscillating fast. Then H'_i reduces to [15]

$$\begin{aligned} H'_i &= \sum_{j=1,2} \frac{g}{2} (e^{-i\delta t} a^+ + e^{i\delta t} a) \sigma_{z,j} \\ &= \frac{g}{2} (e^{-i\delta t} a^+ + e^{i\delta t} a) S_x, \end{aligned} \quad (10)$$

where

$$S_x = \frac{1}{2} \sum_{j=1,2} (S_j^+ + S_j^-) \quad (11)$$

The evolution operator for Hamiltonian H'_i can be written in the form of [16]

$$U'(t) = e^{-iA(t)S_x^2} e^{-iB(t)S_x a} e^{-iC(t)S_x a^+}. \quad (12)$$

Using the Schrödinger equation

$$i \frac{dU'(t)}{dt} = H_i U'(t), \quad (13)$$

we obtain

$$B(t) = \int_0^t \frac{g}{2} e^{i\delta t'} dt' = \frac{g}{2i\delta} (e^{i\delta t} - 1), \quad (14)$$

$$C(t) = \int_0^t \frac{g}{2} e^{-i\delta t'} dt' = \frac{g}{-2i\delta} (e^{-i\delta t} - 1), \quad (15)$$

$$A(t) = i \int_0^t B(t') \frac{g}{2} e^{-i\delta t'} dt' = \frac{g^2}{4\delta} [t + \frac{1}{i\delta} (e^{-i\delta t} - 1)]. \quad (16)$$

Setting

$$\delta t = 2\pi, \quad (17)$$

we have $B(t)=C(t)=0$. Then we have

$$U'(t) = e^{-i\lambda t S_x^2}, \quad (18)$$

where $\lambda = \frac{g^2}{4\delta}$. The evolution operator of the system is given by

$$U(t) = e^{-iH_0 t} U'(t) = e^{-i\Omega t S_x - i\lambda t S_x^2}. \quad (19)$$

It can be easily shown that [17]

$$U(t) |+_1\rangle |+_2\rangle = e^{-i(\Omega+\lambda)t} |+_1\rangle |+_2\rangle,$$

$$U(t) |+_1\rangle |-_2\rangle = |+_1\rangle |-_2\rangle,$$

$$U(t) |-_1\rangle |+_2\rangle = |-_1\rangle |+_2\rangle,$$

$$U(t) |-_1\rangle |-_2\rangle = e^{-i(\Omega-\lambda)t} |-_1\rangle |-_2\rangle. \quad (20)$$

Choose the interaction time t and Rabi frequency Ω appropriately so that

$$\lambda t = \pi/2 \quad (21)$$

and

$$\Omega t = (2k + \frac{1}{2})\pi, \text{ with } k \text{ being an integer.} \quad (22)$$

Then we have

$$U(t) |+_1\rangle |+_2\rangle = -|+_1\rangle |+_2\rangle,$$

$$U(t) |+_1\rangle |-_2\rangle = |+_1\rangle |-_2\rangle,$$

$$U(t) |-_1\rangle |+_2\rangle = |-_1\rangle |+_2\rangle,$$

$$U(t) |-_1\rangle |-_2\rangle = |-_1\rangle |-_2\rangle. \quad (23)$$

By this way we obtain a quantum phase gate. Setting $\delta = g$ and $gt=2\pi$ Eqs. (17) and (21) can be satisfied. We can choose the Rabi frequency Ω appropriately to satisfy (22). In this case the atomic state evolution operator

$U(t)$ is independent of the cavity field state, allowing it to be in a thermal state. In the previous scheme [12], it is required that $\delta \gg g$ and $g^2 t / (4\delta) = \pi$. Therefore, the operation time in the present scheme is much shorter than that in the previous scheme.

We now turn to the problem of generating multiatom entanglement with a thermal cavity. Multiphoton entanglement has been observed and used to verify quantum nonlocality [18]. Schemes have been proposed for producing multiparticle entangled states of hot ions [19,20]. Following the scheme of Ref. [19] four-particle entanglement has been demonstrated in ion traps [21]. On the other hand, three-particle entanglement has been demonstrated in cavity QED [22]. However, in the experiments reported in Refs. [21] and [22] the fidelity of entangled states needs to be significantly improved in order to be useful for the test of quantum nonlocality and in quantum information processing. We have proposed an alternative scheme for the generation of three-atom entangled states with a single cavity always in the vacuum state [23], which is insensitive to the photon decay but sensitive to thermal fields. We here show how we can do so with a single thermal cavity. We consider N identical two-level atoms simultaneously interacting with a single-mode cavity field and driven by a strong classical field. In the case $\delta t = 2\pi$ the evolution operator of the system is given by

$$U(t) = e^{-i\Omega t S_x - i\lambda t S_x^2}, \quad (24)$$

where

$$S_x = \frac{1}{2} \sum_{j=1}^N (S_j^+ + S_j^-). \quad (25)$$

Assume that the atoms are initially in the state $|g_1 g_2 \dots g_N\rangle$. Using the representation of the operator S_z , the atomic state $|g_1 g_2 \dots g_N\rangle$ and $|e_1 e_2 \dots e_N\rangle$ can be expressed as $|N/2, -N/2\rangle$ and $|N/2, N/2\rangle$. On the other hand, such states can be expanded in terms of the eigenstates of S_x [19,24]

$$|N/2, -N/2\rangle = \sum_{M=-N/2}^{N/2} C_M |N/2, M\rangle_x. \quad (26)$$

$$|N/2, N/2\rangle = \sum_{M=-N/2}^{N/2} C_M (-1)^{N/2-M} |N/2, M\rangle_x. \quad (27)$$

Thus, the evolution of the system is

$$\sum_{M=-N/2}^{N/2} C_M e^{-i(\Omega M + \lambda M^2)t} |N/2, M\rangle_x. \quad (28)$$

When N is even M is an integer. With the choice $\lambda t = \pi/2$ and $\Omega t = 2n\pi$ we obtain

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{\sqrt{2}} \sum_{M=-N/2}^{N/2} C_M [e^{-i\pi/4} + e^{i\pi/4} (-1)^M] |N/2, M\rangle_x \\ &= \frac{1}{\sqrt{2}} (e^{-i\pi/4} |N/2, -N/2\rangle + e^{i\pi/4} (-1)^{N/2} |N/2, N/2\rangle). \end{aligned} \quad (29)$$

On the other hand, for the case that N is odd we can set $M = M' + 1/2$, with M' being an integer. With the choice $\lambda t = \pi/2$ and $\Omega t = (4n + \frac{3}{2})\pi$ we obtain

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{\sqrt{2}} e^{i\frac{3}{8}\pi} \sum_{M=-N/2}^{N/2} C_M [e^{-i\pi/4} + e^{i\pi/4} (-1)^{M'}] \\ &\quad |N/2, M\rangle_x \\ &= \frac{1}{\sqrt{2}} e^{i\frac{3}{8}\pi} [e^{-i\pi/4} |N/2, -N/2\rangle \\ &\quad + e^{i\pi/4} (-1)^{(1+N)/2} |N/2, N/2\rangle]. \end{aligned} \quad (30)$$

By this way we obtain multiatom Greenberger-Horne-Zeilinger states [25], which is useful for test of quantum mechanics.

It should be noted that though the atomic state evolution is independent of the thermal photons of cavity field after a period t decided by Eq. (17), the atomic system is entangled with the cavity mode during the atom-cavity interaction. Therefore, it is required that the cavity decay is negligible during the interaction. In the experiment reported in Ref. [14] the atom-cavity coupling strength is about $g = 2\pi \times 50 \text{ kHz}$ and thus the interaction time is on the order of $\pi/g \simeq 10^{-5} \text{ s}$. The photon decay time is $T_c \simeq 10^{-3} \text{ s}$, much longer than the interaction time. After the interaction, the atoms are disentangled with the cavity field and then the cavity decay will not affect the gate operation.

In order to derive Eq. (10) we have assumed that $\Omega \gg \delta, g$ and thus discarded the terms

$$\begin{aligned} \Delta H'_i &= \sum_{j=1,2} \frac{g}{2} [e^{-i\delta t} a^+ (\frac{1}{2} \sigma_j^+ e^{i\Omega t} - \frac{1}{2} \sigma_j^- e^{-i\Omega t}) \\ &\quad + e^{i\delta t} a (\frac{1}{2} \sigma_j^- e^{-i\Omega t} - \frac{1}{2} \sigma_j^+ e^{i\Omega t})]. \end{aligned} \quad (31)$$

These terms induce Stark shifts on the states $|+_j\rangle$ and $|-_j\rangle$. The Stark shifts for $|+_j\rangle$ and $|-_j\rangle$ are on the order of $g^2/(10\Omega)$ and $-g^2/(10\Omega)$, respectively. We here take an example to estimate the error introduced by the Stark shifts. For the generation of two-atom maximally entangled state the fidelity is decreased by $\Delta F_1 \simeq 1 - \frac{1}{4} \{1 + \cos[g^2 t / (5\Omega)]\}^2$. Setting $\Omega = 5\delta$, We have $\Delta F_1 \simeq 0.03$.

The present scheme requires that two atoms be simultaneously sent through a cavity, otherwise there will be an error. Assume that during the generation of two-atom maximally entangled state one atom enters the cavity $0.01t$ sooner than another atom. In this case the fidelity is decreased by $\Delta F_2 \simeq \sin^2(0.01\Omega t/2) + \sin^2(0.91\lambda t) \simeq$

0.02. Suppose the fluctuation of the Rabi frequency Ω is $\Delta\Omega = 0.01\Omega$. This fluctuation also decreases the fidelity by $\Delta F_3 \simeq \sin^2(0.01\Omega t/2) \simeq 0.02$.

In conclusion, we have proposed a scheme for realizing two-qubit phase gates in cavity QED. The scheme is insensitive to the thermal field and works in a fast way, which is of importance from the experimental point of view. Our scheme may offer a viable way to build a scalable quantum computer. Our scheme can also be used to

produce multiatom entangled states with a single thermal cavity.

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- [1] P. W. Shor, in Proceedings of the 35th Annual Symposium on Foundations of Computer Science (IEEE Computer Society, Los Alamitos, CA, 1994), P.116.
 - [2] L. K. Grover, Phys. Rev. Lett. 79 (1997) 325; *ibid.* 79 (1997) 4709.
 - [3] T. Sleator and H. Weinfurter, Phys. Rev. Lett. 74, 4087 (1995).
 - [4] A. Barenco et al., Phys. Rev. Lett. 74, 4083 (1995); P. Domokos et al., Phys. Rev. A 52, 3554 (1995); T. Pellizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. 75, 3788 (1995).
 - [5] J. I. Cirac and P. Zoller, Phys. Rev. Lett. 74, 4091 (1995); C. Monroe et al., Phys. Rev. A 55, R2489 (1997).
 - [6] Q. A. Turchette et al., Phys. Rev. Lett. 75, 4710 (1995).
 - [7] A. Rauschenbeutel et al., Phys. Rev. Lett. 83, 5166 (1999).
 - [8] C. Monroe et al., Phys. Rev. Lett. 75, 4714 (1995).
 - [9] N.A. Gershenfeld and I. L. Chuang, Science 275, 350 (1997); D. G. Cory et al., Proc. Natl. Acad. Sci. U. S. A. 94, 1634 (1997); J.A. Jones et al., Nature (London) 393, 344 (1998).
 - [10] A. Sørensen and K. Mølmer, Phys. Rev. Lett. 82, 1971 (1999).
 - [11] D. Jonathan and M. B. Plenio, Phys. Rev. Lett. 87, 127901 (2001).
 - [12] S. B. Zheng and G. C. Guo, Phys. Rev. Lett. 85, 2392 (2000).
 - [13] S. B. Zheng and G. C. Guo, Phys. Rev. A 63, 044302 (2001).
 - [14] S. Osnaghi et al., Phys. Rev. Lett. 87, 037902 (2001).
 - [15] E. Solano et al., quant-ph/0202071.
 - [16] A. Sørensen and K. Mølmer, Phys. Rev. A 62, 022311 (2000).
 - [17] X. Zou et al., quant-ph/0204171.
 - [18] J-W. Pan et al., *Nature* **403**, 515 (2000).
 - [19] K. Mølmer and A. Sørensen, Phys. Rev. Lett. 82, 1835 (1999).
 - [20] S. B. Zheng, Phys. Rev. A 65, 051804 (R) (2002).
 - [21] C. A. Sackett et al., *Nature* **404**, 256 (2000).
 - [22] A. Rauschenbeutel et al., Science 288, 2024 (2000).
 - [23] S. B. Zheng, Phys. Rev. Lett. 87, 230404 (2001).
 - [24] D. M. Brink and G. R. Satchler, Angular Momentum (Clarendon Press, Oxford, 1975).
 - [25] D. M. Greenberger et al., in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe*, edited by M.Kafatos (Kluwer, Dordrecht, 1989); D. M. Greenberger et al., Am. J. Phys. 58, 1131 (1990).